Formal algebraic description of a Fog Computing environment

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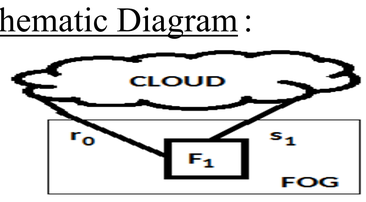
Introduction

• Fog Computing is an evolution from Cloud Computing. The development of Fog Computing facilities is key for the full scale deployment of IoT environments, where VMs may move from host to host following the moves of their associated users.

•We want to achieve a formal algebraic general expression, by means of an abstract algebra called ACP, for the movement of VMs belonging to users getting from the Cloud into a Fog environment, composed by a linear distribution of hosts (building blocks), such as a long avenue or a pipeline.

1 Building Block

▪ Schematic Diagram:



▪Model description in ACP

$$F_1 = r_0 \cdot s_1 \cdot F_1$$

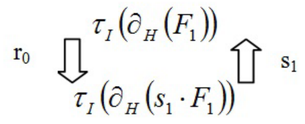
▪Applying the encapsulation operator

$$\partial_H(F_1) = r_0 \cdot \partial_H(s_1 \cdot F_1)$$
$$\partial_H(s_1 \cdot F_1) = s_1 \cdot \partial_H(F_1)$$

▪Applying the abstraction operator

$$\tau_I(\partial_H(F_1)) = r_0 \cdot \tau_I(\partial_H(s_1 \cdot F_1))$$
$$\tau_I(\partial_H(s_1 \cdot F_1)) = s_1 \cdot \tau_I(\partial_H(F_1))$$

▪State Diagram:



▪Real system external behaviour

➢X→ 0 items inside the block

➢Y→ 1 item inside the block

$$X = r_0 \cdot Y$$
$$Y = s_1 \cdot X$$

▪Real system –vs– model behavior

➢State matching: X and $\tau_I(\partial_H(F_1))$

➢State matching: Y and $\tau_I(\partial_H(s_1 \cdot F_1))$

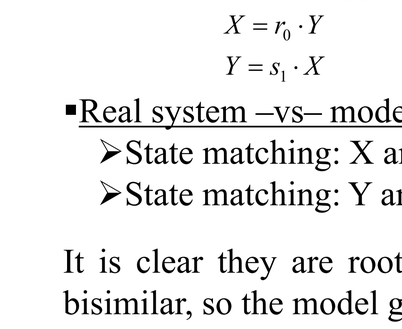
It is clear they are rooted branching bisimilar, so the model gets verified

N Building Blocks

•The range of all external states in the system goes from 0 to N (N+1 states)

•Also, for each external k-th state ($0 \le k \le N$), there is a number of equivalent expressions, denoting all its possible positions into the system (its equivalent internal states)

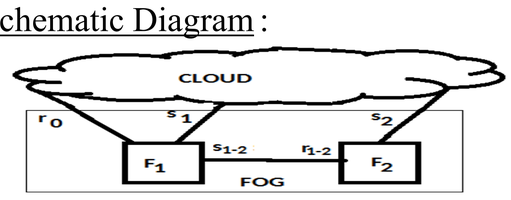
▪Those numbers are given by the n-th row and the k-th column of the Pascal Triangle, or alternatively, by the number of combinations available



$$C(n,k) = \binom{n}{k} = \frac{n!}{k!\cdot(n-k)!}$$

2 Building Blocks

▪ Schematic Diagram:



▪Model description in ACP

$$F_1 = r_0 \cdot (s_1 + s_{1\to 2}) \cdot F_1$$
$$F_2 = r_{1\to 2} \cdot s_1 \cdot F_2$$

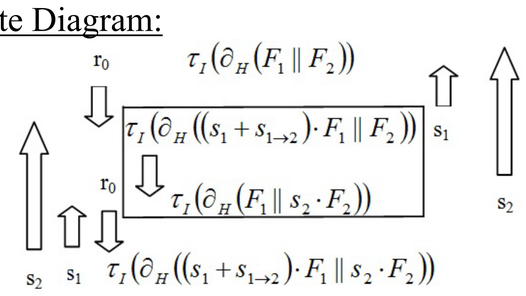
▪Applying the encapsulation operator

$$\partial_H(F_1 \parallel F_2) = r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2) = s_1 \cdot \partial_H(F_1 \parallel F_2) + c_{1\to 2} \cdot \partial_H(F_1 \parallel s_2 \cdot F_2)$$
$$\partial_H(F_1 \parallel s_2 \cdot F_2) = r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel s_2 \cdot F_2) + s_2 \cdot (F_1 \parallel F_2)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel s_2 \cdot F_2) = s_1 \cdot \partial_H(F_1 \parallel s_2 \cdot F_2) + s_2 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2)$$

▪Applying the abstraction operator

$$\tau_I(\partial_H(F_1 \parallel F_2)) = r_0 \cdot \tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2))$$
$$\tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2)) = s_1 \cdot \tau_I(\partial_H(F_1 \parallel F_2)) + \tau_I(\partial_H(F_1 \parallel s_2 \cdot F_2))$$
$$\tau_I(\partial_H(F_1 \parallel s_2 \cdot F_2)) = r_0 \cdot \tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel s_2 \cdot F_2)) + s_2 \cdot \tau_I(\partial_H(F_1 \parallel F_2))$$
$$\tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel s_2 \cdot F_2)) = s_1 \cdot \tau_I(\partial_H(F_1 \parallel s_2 \cdot F_2)) + s_2 \cdot \tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2))$$

▪State Diagram:



▪Real system external behaviour

➢X→ 0 items inside the block

➢Y→ 1 item inside the block

•There may be 1 in the 1st or in the 2nd block

➢Z→ 2 items inside the block

$$X = r_0 \cdot Y$$
$$Y = r_0 \cdot Z + (s_1 + s_2) \cdot X$$
$$Z = (s_1 + s_2) \cdot Y$$

▪Real system –vs– model behavior

➢State matching: X and $\tau_I(\partial_H(F_1 \parallel F_2))$

➢State matching: Y and $\tau_I(\partial_H(F_1 \parallel s_2 \cdot F_2))$

•Transition from 1st or in the 2nd block is internal

➢State matching: Z and $\tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel s_2 \cdot F_2))$

It is clear they are rooted branching bisimilar, so the model gets verified

3 Logic Rules

$$I: \frac{r_0 \to V}{\partial_H(F_1 \parallel \ldots \parallel F_N) \to \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel \ldots \parallel F_N)}$$

$$II: \frac{c_{i\to i+1,(i\in[1,N-1])} \to V}{\partial_H(\ldots \parallel (s_i + s_{i\to i+1}) \cdot F_i \parallel F_{i+1} \parallel \ldots) \to \partial_H(\ldots \parallel F_i \parallel (s_{i+1} + s_{i+1\to i+2}) \cdot F_{i+1} \parallel \ldots)}$$

$$III: \frac{s_{i,(i\in[1,N])} \to V}{\partial_H(\ldots \parallel (s_i + s_{i\to i+1}) \cdot F_i \parallel \ldots) \to\to \partial_H(\ldots \parallel F_i \parallel \ldots)}$$

$X_0 = 0 \longrightarrow \tau_I(\partial_H(F_1 \parallel \ldots \parallel F_{N-1} \parallel F_N))$

$X_1 = 1 \longrightarrow \tau_I(\partial_H(F_1 \parallel \ldots \parallel F_{N-1} \parallel s_N \cdot F_N))$

$X_2 = 2 \longrightarrow \tau_I(\partial_H(F_1 \parallel \ldots \parallel (s_{(N-1)\to N} + s_{N-1}) \cdot F_{N-1} \parallel s_N \cdot F_N))$

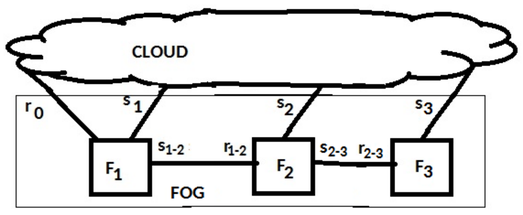
…….

i-th state (0 ≤ i ≤ N)

$$X_i = i \longrightarrow \tau_I(\partial_H(F_1 \parallel \ldots \parallel (s_{(N-i)\to(N-i+1)} + s_{N-i}) \cdot F_{N-i} \parallel \ldots \parallel s_N \cdot F_N))$$

3 Building Blocks

▪ Schematic Diagram:



▪Model description in ACP

$$F_1 = r_0 \cdot (s_1 + s_{1\to 2}) \cdot F_1$$
$$F_2 = r_{1\to 2} \cdot (s_2 + s_{2\to 3}) \cdot F_2$$
$$F_3 = r_{2\to 3} \cdot s_3 \cdot F_3$$

▪Applying the encapsulation operator

$$\partial_H(F_1 \parallel F_2 \parallel F_3) = r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel F_3)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel F_3) = s_1 \cdot \partial_H(F_1 \parallel F_2 \parallel F_3) + c_{1\to 2} \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3)$$
$$\partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) = s_2 \cdot \partial_H(F_1 \parallel F_2 \parallel F_3) + c_{2\to 3} \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) + r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3)$$
$$\partial_H(F_1 \parallel F_2 \parallel s_3 \cdot F_3) = s_3 \cdot \partial_H(F_1 \parallel F_2 \parallel F_3) + r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel s_3 \cdot F_3)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) = s_1 \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) + s_2 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel F_3) + c_{2\to 3} \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel s_3 \cdot F_3)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel s_3 \cdot F_3) = s_1 \cdot \partial_H(F_1 \parallel F_2 \parallel s_3 \cdot F_3) + s_3 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel F_3) + c_{1\to 2} \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3)$$
$$\partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3) = s_2 \cdot \partial_H(F_1 \parallel F_2 \parallel s_3 \cdot F_3) + s_3 \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) + r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3)$$
$$\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3) = s_1 \cdot \partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3) + s_2 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel s_3 \cdot F_3) + s_3 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3) + r_0 \cdot \partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3)$$

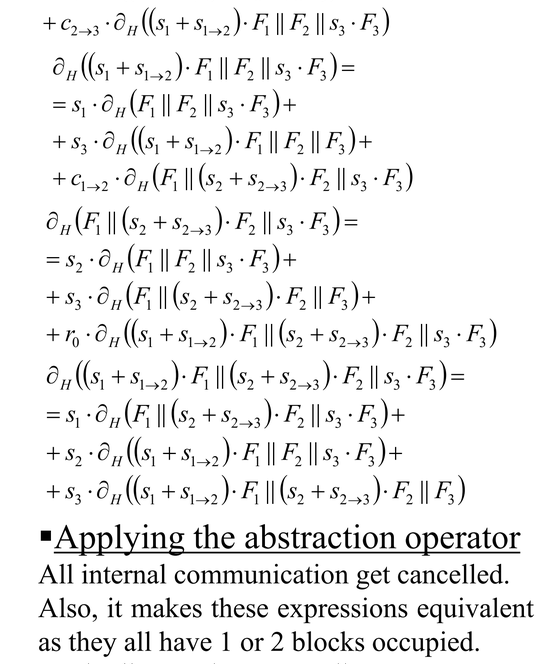
▪Applying the abstraction operator

All internal communication get cancelled. Also, it makes these expressions equivalent as they all have 1 or 2 blocks occupied.

$$\tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel F_3)) \Leftrightarrow$$
$$\Leftrightarrow \tau_I(\partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3)) \Leftrightarrow$$
$$\Leftrightarrow \tau_I(\partial_H(F_1 \parallel F_2 \parallel s_3 \cdot F_3))$$

$$\tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel F_3)) \Leftrightarrow$$
$$\Leftrightarrow \tau_I(\partial_H((s_1 + s_{1\to 2}) \cdot F_1 \parallel F_2 \parallel s_3 \cdot F_3)) \Leftrightarrow$$
$$\Leftrightarrow \tau_I(\partial_H(F_1 \parallel (s_2 + s_{2\to 3}) \cdot F_2 \parallel s_3 \cdot F_3))$$

▪State Diagram:



▪Real system external behaviour

➢X_i→i items inside the block

$$X_0 = r_0 \cdot X_1$$
$$X_1 = r_0 \cdot X_2 + (s_1 + s_2 + s_3) \cdot X_0$$
$$X_2 = r_0 \cdot X_3 + (s_1 + s_2 + s_3) \cdot X_1$$
$$X_3 = (s_1 + s_2 + s_3) \cdot X_2$$

▪Real system –vs– model behavior

It is clear that the model gets verified