Optimal Design of Linear Consecutive Systems

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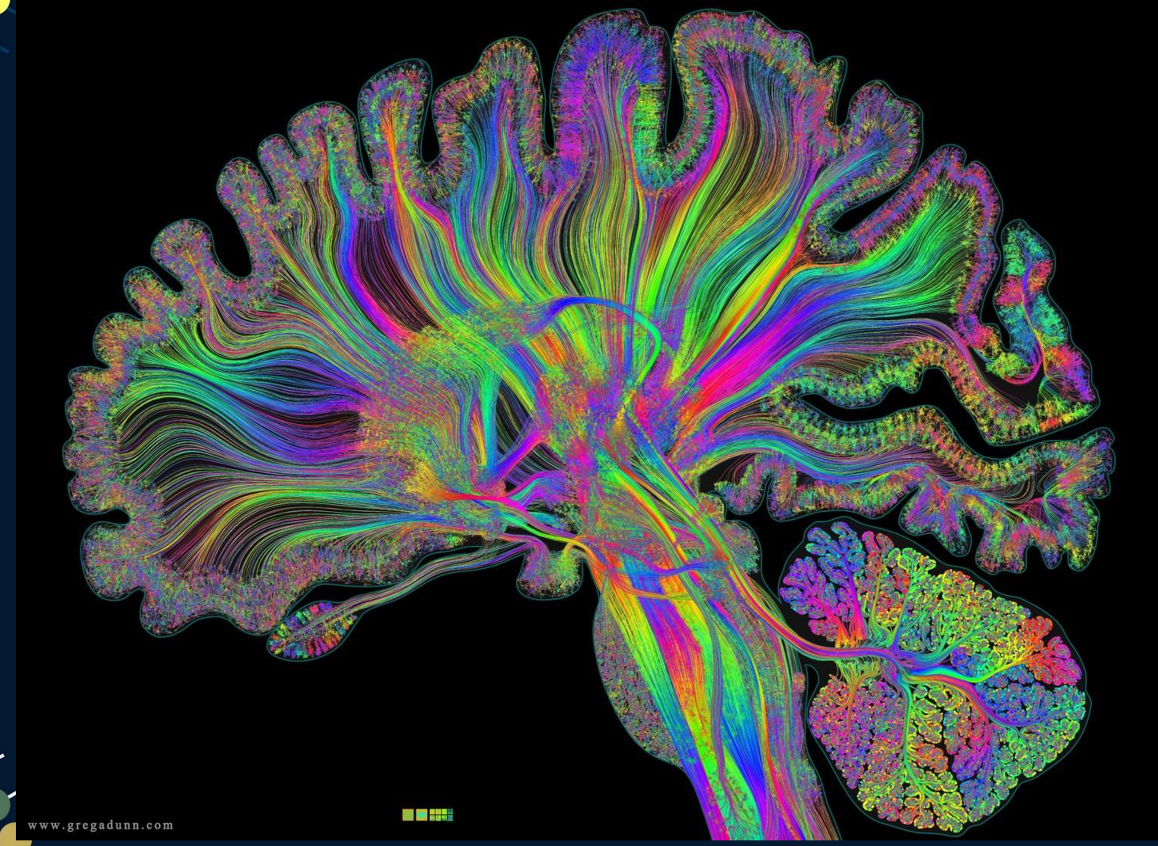
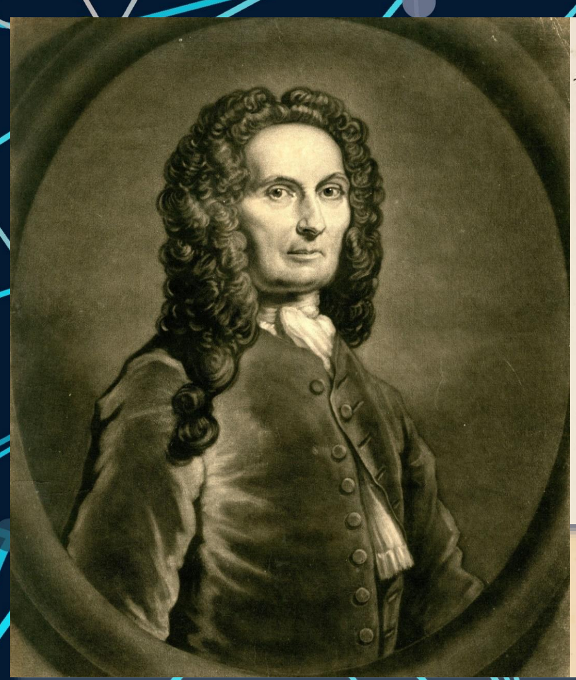
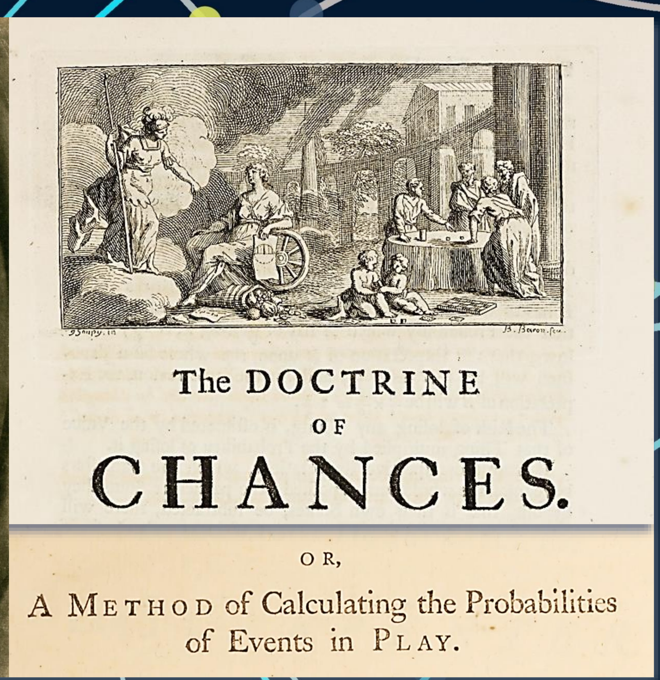
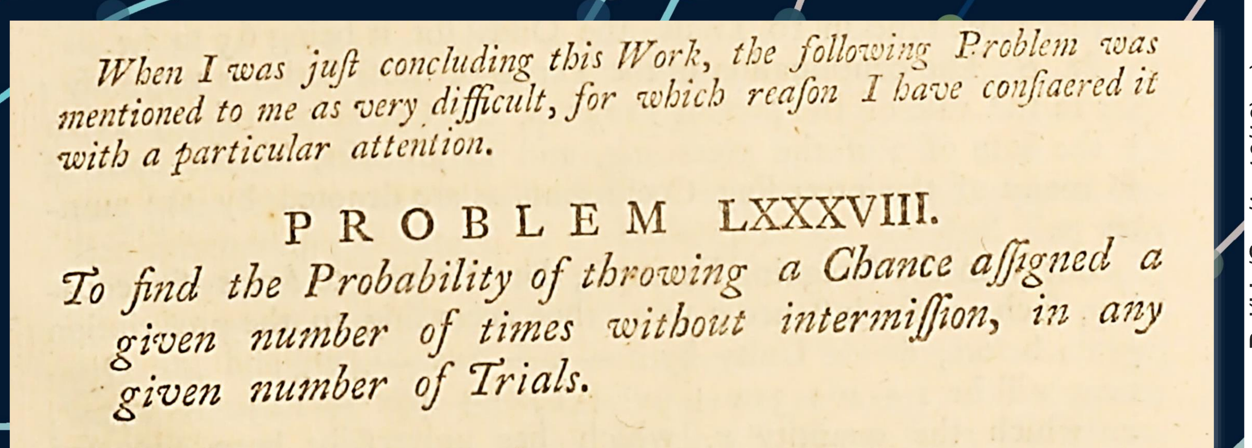


Neurons are prime examples of efficiency, achieving outstanding **communication reliabilities**, although relying on random ion channels. Aiming to bridge from biology to circuits, we show here how statistical results about linear consecutive systems, combined with a Binet-like formula for Fibonacci numbers of higher orders, lead to trivial reliability calculations for neuron-inspired optimal design schemes for **communication**.





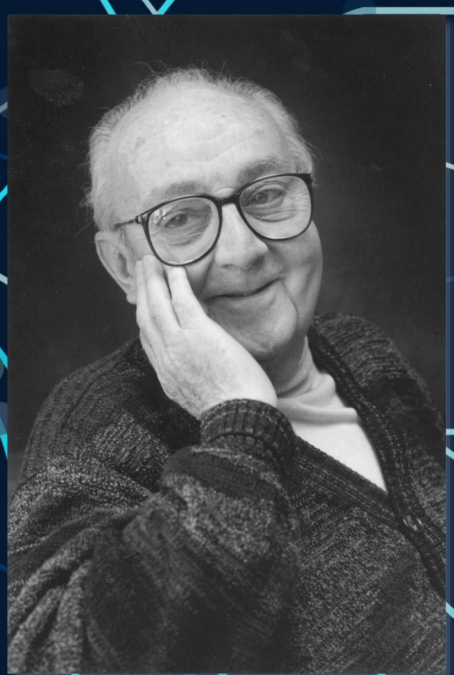
The DOCTRINE OF CHANCES. OR, A METHOD of Calculating the Probabilities of Events in PLAY.

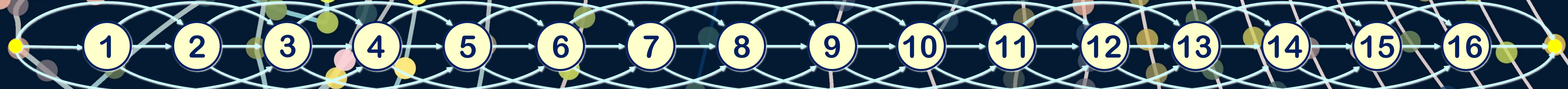


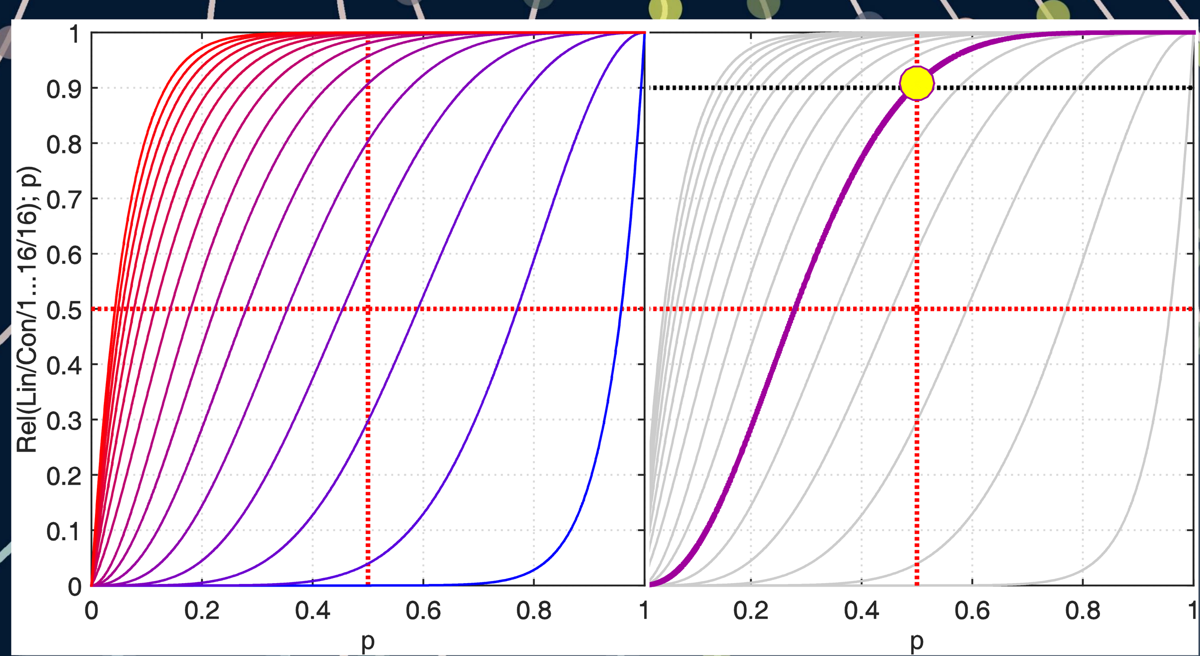
When I was just concluding this Work, the following Problem was mentioned to me as very difficult, for which reason I have considered it with a particular attention.

PROBLEM LXXXVIII.

To find the Probability of throwing a Chance assigned a given number of times without intermission, in any given number of Trials.







- [...] all models are approximations
- **Essentially, all models are wrong, but some are useful**
- [...] the approximate nature of the model must always be borne in mind

$$Rel(Lin/Con/k/n;0.5) = \frac{F_{n+2}^{(k)}}{2^n}$$ Philippou 1985

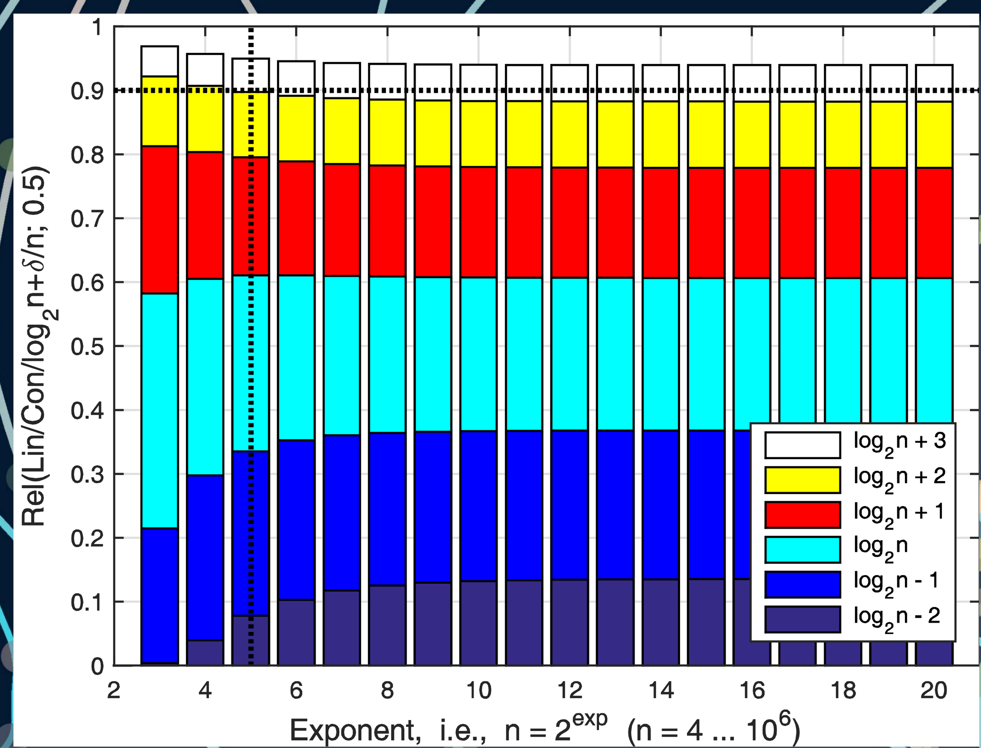
$$F_n^{(k)} = F_{n-1}^{(k)} + F_{n-2}^{(k)} + \cdots + F_{n-k}^{(k)}$$ d'Ocagne 1884

$$x^k - x^{k-1} - x^{k-2} - \cdots - x^2 - x - 1 = 0$$

$$\alpha_k = 2\left[1 - \sum_{i\geq 1}\frac{1}{i}\binom{(k+1)i-2}{i-1}\frac{1}{2^{(k+1)i}}\right]$$ Wolfram 1998

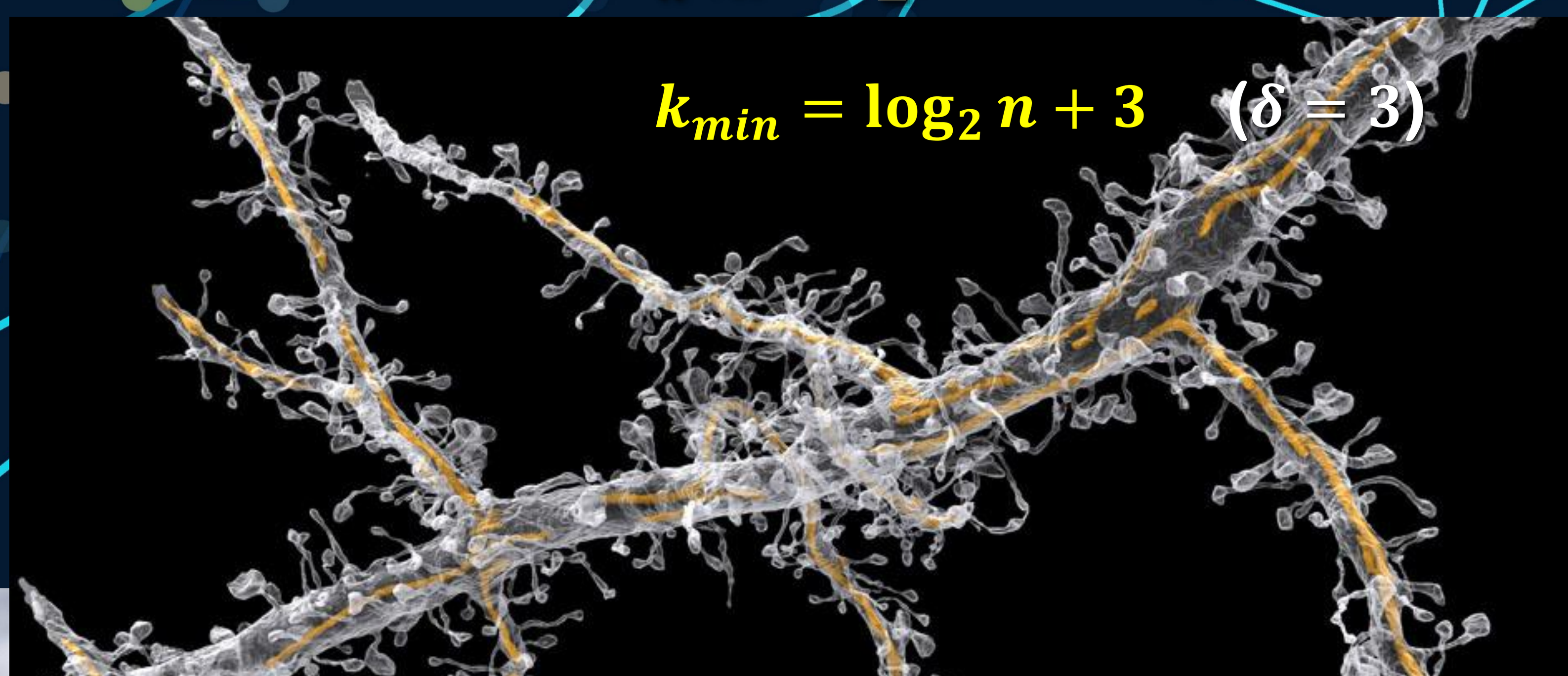
$$2 - \frac{1}{2^{k-1}} < 2 - \frac{1}{2^{k-\frac{k}{2}-\frac{k^2}{2^k}}} < \alpha_k < 2 - \frac{1}{2^{k-\frac{k}{2}}} < 2 - \frac{1}{2^k}$$

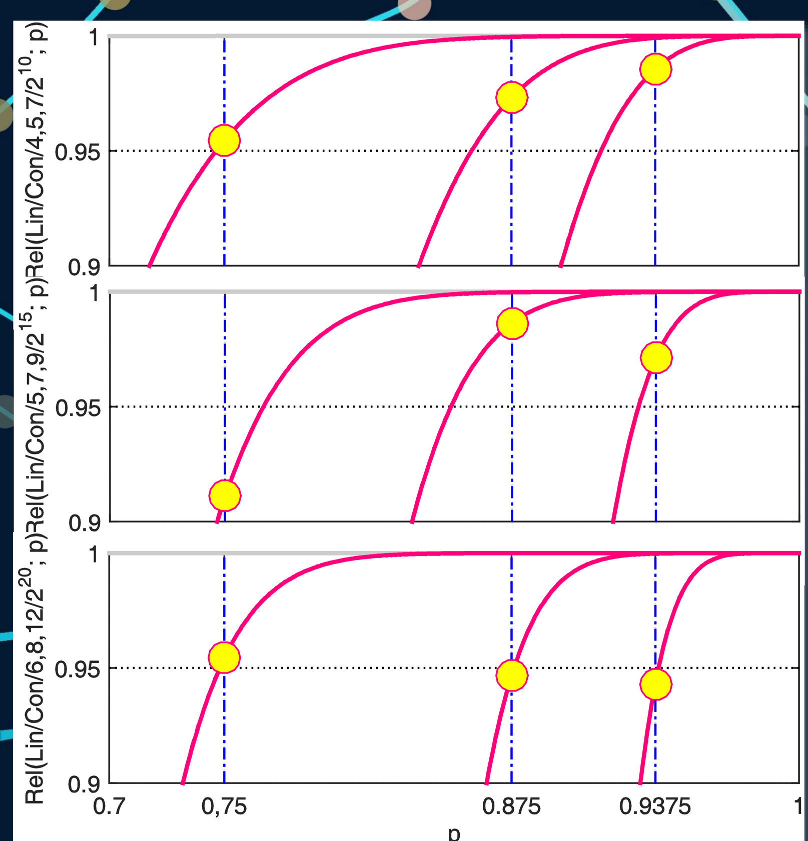
$$F_n^{(k)} = \mathrm{rnd}\left[\frac{\alpha_k - 1}{2+(k+1)(\alpha_k-2)}\alpha_k^{n-1}\right]$$ Du 2008, Dresden 2009



$$\lim_{n\to\infty}\frac{F_{n+2}^{(\log_2 n+\delta)}}{2^n} = e^{-1/2^{1+\delta}}$$

$$k_{min} = \log_2 n + 3 \quad (\delta = 3)$$

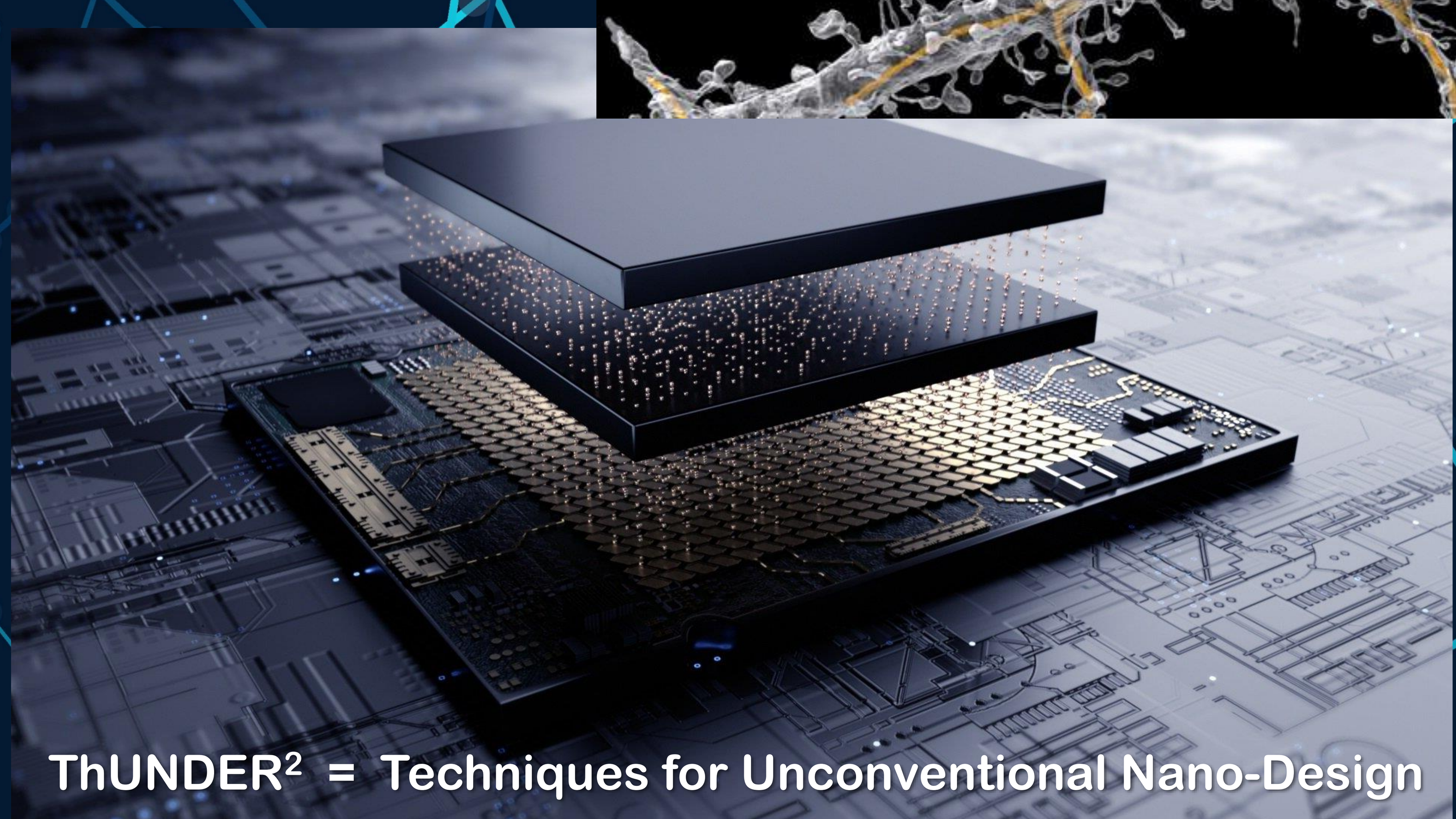




Axons correspond to 2D consecutive systems:

- (m,k)-out-of-(m,n)
- $n = 10^2 \ldots 10^6$
- $m = 2, 3, 4$

$$k_{min} = \left\lceil\frac{\log_2 n + 3}{m}\right\rceil$$



Our method for optimizing linear consecutive systems:

- avoids computing the reliability polynomial
- relies (trivially) on a Binet-like formula
- $Rel > 90\%$ is achieved for any $n > 4$
- has just been extended to 2D (axons)
- technology mapping is underway

THANK YOU

ThUNDER[2] = Techniques for Unconventional Nano-Design … in the Energy-Reliability Realm PN-III-P4-ID-PCE-2020-2495 nr. 238/07.04.2021