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An active learning-oriented error-based stopping criterion for efficient structural reliability analysis

Highlights

- An error-based stopping criterion derived from Chebyshev's inequality without entailing any assumption is proposed to terminate the active learning process.
- A hybrid stopping criterion is developed by jointly considering the failure probability estimation error and its stabilization property at the convergence stage.
- The proposed hybrid algorithm is found to be able to more efficiently achieve desirable accuracy than its counterparts.
- This active learning-oriented stopping criterion can be integrated with any surrogate-based method for efficient reliability analysis.

Introduction

The reliability analysis of complex engineering structures usually involves implicit performance functions and time-demanding simulation models, hence reducing the required number of functional calls is crucial to improving computational efficiency. In this regard, the surrogate-based active learning reliability analysis methods (as shown in Fig. 1) have gained extensive attention for their capability to achieve an excellent trade-off between accuracy and efficiency.

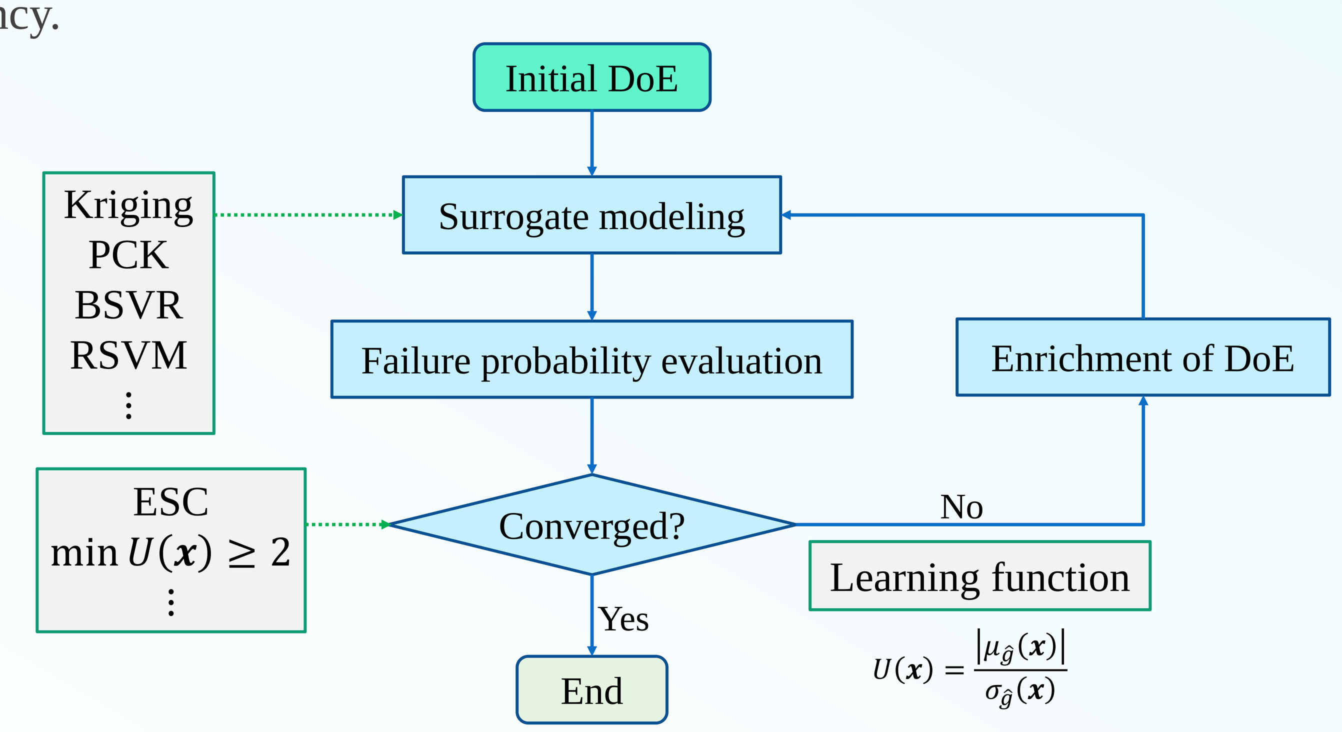


Fig. 1. The surrogate-based active learning method for reliability analysis.

In the active learning methods, the learning function and the stopping criterion are two of the most essential ingredients that influence the overall performance of the algorithm. The focus of this work is to develop an efficient algorithm to terminate the active learning process at an appropriate stage. Specifically, an error-based stopping criterion (ESC) is derived according to Chebyshev's inequality, whereby the upper bound of the failure probability estimation error can be easily calculated without entailing any assumption or the bootstrap resampling analysis (BESC). Thereafter, a hybrid stopping criterion that accounts for the failure probability estimation error and its stabilization property at the converged stage is developed to enhance the computational efficiency of active learning methods. For illustration purposes, the proposed stopping criterion is integrated with the Bayesian support vector regression (BSVR) [1,2] to form the ABSVR method for efficient structural reliability analysis.

The ESC derived from Chebyshev's inequality (CESC)

The error-based stopping criterion (ESC) was originally developed in [1], and can be expressed as:

$$\epsilon_r \leq \max\left(\left|\frac{\widehat{N}_f}{\widehat{N}_f - \widehat{N}_{fs}^u} - 1\right|, \left|\frac{\widehat{N}_f}{\widehat{N}_f + \widehat{N}_{sf}^u} - 1\right|\right) = \hat{\epsilon}_{\max} \leq \epsilon_{tol}$$

where $\hat{\epsilon}_{\max}$ is the upper bound of the failure probability error ϵ_r; ϵ_{tol} is the threshold value.

The key for ESC is to obtain the confidence interval of the number of samples ($\widehat{N}_{fs}$ and $\widehat{N}_{sf}$) whose sign is being wrongly predicted by a surrogate model. To date, the confidence interval can be derived according to the probabilistic properties of Poisson binomial distribution [3] or using the bootstrap resampling method (BESC) [4]. These two approaches either involve assumptions that might be invalid for a small sample size or entail additional computational costs. In this study, Chebyshev's inequality is adopted to derive the confidence interval of $\widehat{N}_{fs}$ and $\widehat{N}_{sf}$ [5].

Simple is beautiful!

$$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2} \longrightarrow \begin{cases} P\left(\mu - \frac{\sigma}{\sqrt{1-\gamma}} < X < \mu + \frac{\sigma}{\sqrt{1-\gamma}}\right) > \gamma \\ \gamma = 1 - \frac{1}{k^2}, \quad 0 \leq \gamma < 1 \end{cases} \longrightarrow \begin{cases} \widehat{N}_{sf}^u = \mu_{\widehat{N}_{sf}} + \frac{\sigma_{\widehat{N}_{sf}}}{\sqrt{1-\gamma}} \\ \widehat{N}_{fs}^u = \mu_{\widehat{N}_{fs}} + \frac{\sigma_{\widehat{N}_{fs}}}{\sqrt{1-\gamma}} \end{cases}$$

A hybrid stopping criterion based on ESC (HESC)

For some problems, the accuracy of the failure probability estimation is found to have been stabilized before the ESC is met. Thus, a hybrid stopping criterion that can exploit the stabilization property of the failure probability is proposed as follows:

1. $$\begin{cases} \left|\frac{\hat{P}_f^i - \hat{P}_f^{i-1}}{\hat{P}_f^{i-1}}\right| \leq \epsilon_{tol1} \text{ and } \left|\frac{\hat{P}_f^{i-1} - \hat{P}_f^{i-2}}{\hat{P}_f^{i-2}}\right| \leq \epsilon_{tol1}, i \geq 3 \\ \max\left(\left|\frac{\widehat{N}_f}{\widehat{N}_f - \widehat{N}_{fs}^u} - 1\right|, \left|\frac{\widehat{N}_f}{\widehat{N}_f + \widehat{N}_{sf}^u} - 1\right|\right) = \hat{\epsilon}_{\max} \leq \epsilon_{tol2} \end{cases}$$

2. $$\max\left(\left|\frac{\widehat{N}_f}{\widehat{N}_f - \widehat{N}_{fs}^u} - 1\right|, \left|\frac{\widehat{N}_f}{\widehat{N}_f + \widehat{N}_{sf}^u} - 1\right|\right) = \hat{\epsilon}_{\max} \leq \epsilon_{tol}$$

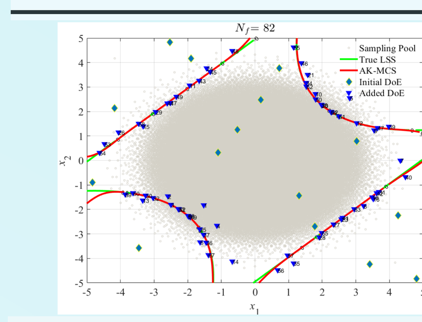
in which $\epsilon_{tol} < \epsilon_{tol2}$, and is set as $\epsilon_{tol} = 0.01$, $\epsilon_{tol2} = 0.1$ in this study; $\epsilon_{tol1} = 0.001$.

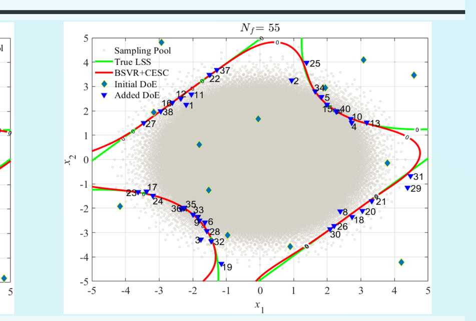
Numerical examples

A series system with four branches

$$g(\boldsymbol{x}) = \min\begin{Bmatrix} 3 + 0.1(x_1 - x_2)^2 - \frac{x_1 + x_2}{\sqrt{2}} \\ 3 + 0.1(x_1 - x_2)^2 + \frac{x_1 + x_2}{\sqrt{2}} \\ (x_1 - x_2) + \frac{7}{\sqrt{2}} \\ (x_2 - x_1) + \frac{7}{\sqrt{2}} \end{Bmatrix}$$

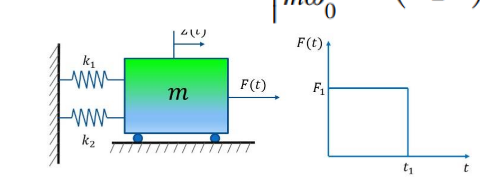
Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	2.221×10^{-3}	2.845	1×10^7	–
AK-MCS	2.235×10^{-3}	2.843	84.9	0.63
ESC+U	2.229×10^{-3}	2.844	57.8	0.36
BESC+U	2.215×10^{-3}	2.846	57.5	0.27
CESC+U	2.226×10^{-3}	2.844	58.6	0.23
HESC+U	2.214×10^{-3}	2.845	55.2	0.32





A nonlinear oscillator

$$g(c_1, c_2, m, r, t_1, F_1) = 3r - \left|\frac{2F_1}{m\omega_0^2}\sin\left(\frac{\omega_0 t_1}{2}\right)\right|$$

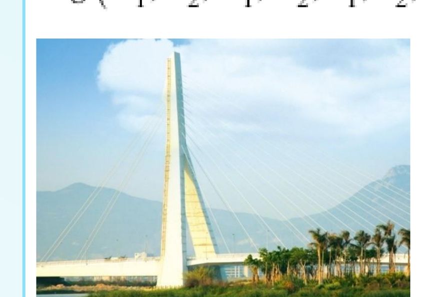


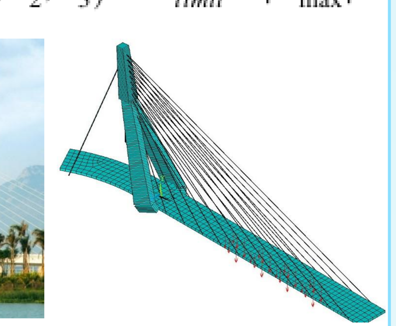
Random variable	Distribution	Mean	Standard deviation
m	Normal	1	0.05
c_1	Normal	1	0.1
c_2	Normal	0.1	0.01
r	Normal	0.5	0.05
t_1	Normal	1	0.2
F_1	Normal	1	0.2

Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	2.859×10^{-3}	1.902	1×10^7	–
AK-MCS	2.852×10^{-3}	1.903	530	0.24
ESC+U	2.863×10^{-3}	1.902	55.8	0.14
BESC+U	2.856×10^{-3}	1.902	52.7	0.10
CESC+U	2.861×10^{-3}	1.901	54.5	0.07
HESC+U	2.854×10^{-3}	1.902	46.9	0.17

A single tower cable-stayed bridge

$$g(E_1, E_2, D_1, D_2, F_1, F_2, F_3) = \Delta_{limit} - |\Delta_{\max}|$$





Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	6.732×10^{-2}	1.496	1×10^5	–
AK-MCS	6.804×10^{-2}	1.491	190.8	1.07
ESC+U	6.791×10^{-2}	1.491	53.5	0.88
BESC+U	6.663×10^{-2}	1.500	55.5	1.02
CESC+U	6.765×10^{-2}	1.490	54.0	0.49
HESC+U	6.800×10^{-2}	1.489	40.3	1.01

Conclusions

1. The ESC derived from Chebyshev's inequality (CESC) can provide results with comparable accuracy and efficiency as its counterparts, i.e., ESC and BESC.
2. The proposed hybrid stopping criterion (HESC) is generally more efficient than its ESC-based counterparts, without compromising the estimation accuracy.
3. The proposed criterion is simple and entails no assumptions nor bootstrap analysis; can be integrated with any surrogate-based method for efficient reliability analysis.

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